Yellow Difficulty Level MathCat

Problem 1. (6 credits) There are 14 people in a room. Each of them is either a knight (who makes only true statements) or a liar (who makes only false statements). Each one declared, "How awful! There are more liars than knights among those I see here!" How many liars are in the room?

Problem 2. (6 credits) All natural numbers having any two adjacent digits of different parity and the sum of their digits equal to 57 are written in ascending order. Which number is written second?

Problem 3. (8 credits) Two teams of children shared some candies. The captain of the first team told that they had received 4/7 of all candies, and the captain of the second team told that they had received 1/2 of all candies. But suddenly it turned out that both captains thought that Wendy was on their team, and Peter was not. Wendy received 8 candies, and Peter received only 3 candies. How many candies were there in total?

Problem 4. (8 credits) All squares of an 11x11 chessboard are painted white and black in a checkerboard pattern. All corners of the board are black. In how many ways can one red chip and one green chip be placed on the black squares being placed at the same time on the same horizontal or vertical line?

Problem 5. (10 credits) A five-digit number can be broken into a three-digit number and a two-digit number in two ways. In one of these cases, the sum of its parts is 168, in the other case the sum is 357. Find the initial five-digit number and specify all possible solutions.

Problem 6. (10 credits) 14 gentlemen are members of a club. Over the course of a week, each member met other member once. Each time they met, the first gentleman asked the second member to pass on his greetings to all other members this second gentleman might see again. How many greetings were passed on in total?

Problem 7. (12 credits) Let $1 = d_1 < d_2 < ... < d_k = n$ (*"d"* are divisors of the natural number n).

It turned out that $d_2^2 + d_3^2 = n + 4$. Find n.

Problem 8. (12 credits) In the triangle ABC, which is not isosceles, ∠A = 40°. A point moved along the bisector of the angle A, reflected from the side BC, and stopped at the same distance from the points B and C. At what angle is the side BC visible from this point?
Additional information:The angle of incidence of the point equals the angle of its reflection. It is considered that the segment XY is visible from the point *T* at the angle *XTY.*

Problem 9. (14 credits) A mathematician calculated the sum $9 + 99 + 999 + ... + \underbrace{99...99}_{\text{2025 times}}$ and wrote it on the blackboard. How many times does the digit '1' appear in this result?

Problem 10. (14 credits) A binary operation ∗ assigns to every ordered pair of numbers a,b a number denoted by a∗b, such that for all a and b the following conditions hold true:
(1) $(a * b) + (b * a) = 2$,
(2) $(a + 1) * b = (a * b) + 3$.
Find the value of 1*2023.